

Lab 23

Spectrum of Atomic Hydrogen and Planck Constant

A. Purpose

To investigate the spectrum of hydrogen atom, determine the Planck constant by the Balmer series, and find the index of refraction of a prism using a spectrometer.

B. Introduction

A spectrometer is a scientific instrument that disperses a beam of light with a prism or diffraction grating. If the beam is composed of more than one color, a spectrum is formed since various colors are refracted or diffracted to different angles. When an electron changes its orbit within an atom, a light of a specific wavelength will be emitted or absorbed. Because of this, a spectrometer is a powerful tool for investigating the structure of atoms and molecules, and astronomers can also determine the compositions of the sun and stars. In this lab, we will utilize a spectrometer to observe the Balmer series in the hydrogen spectrum and determine the Planck constant. Moreover, we will determine the index of refraction of a prism using the angle of minimum deviation.

The hydrogen atoms in a discharge lamp emit four visible spectrum lines, whose wavelengths are 410.2 nm (violet line), 434.1 nm (blue line), 483.1 nm (green line), and 656.3 nm (red line). Empirically, in 1885, Balmer developed a mathematical relation that connects these lines. The formula is given by

$$\lambda = B \frac{n^2}{n^2 - 2^2} \quad n = 3, 4, 5, \dots \quad (1)$$

where λ is the wavelength of the lines and $B = 364.6 \text{ nm}$ is known as Balmer's constant. In 1889, Johann Rydberg empirically found a more generalized formula that can be used to connect all lines in the emission spectrum of hydrogen atoms.

$$\frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \quad n_i > n_f \quad (2)$$

where $R = 1.097 \times 10^7 \text{ m}^{-1}$ is called Rydberg constant, and n_i and n_f are integers (later known as principle quantum numbers) with $n_i > n_f$. For the hydrogen atom, $n_f = 2$ corresponds to the Balmer series.

This empirical formula was explained by the solar-system-like model proposed by Niels Bohr in 1913. The Bohr model was a major step to the development of Quantum Mechanics. In this model, Bohr postulated

1. The electron circulates the nucleus due to the Coulomb force

$$F = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} = \frac{mv^2}{r} \tag{3}$$

where F is the centripetal force for the circulating electron of mass m and velocity v in the orbit of radius r and $\epsilon_0 = 8.85 \times 10^{-12} \text{ F} \cdot \text{m}^{-1}$ is the vacuum permittivity.

2. Unlike in classical mechanics, the electron is only allowed to move in the orbits of quantized angular momentums

$$L = mvr = n \frac{h}{2\pi} = n\hbar, \quad n = 1, 2, 3, \dots \tag{4}$$

where L is the angular momentum, $h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$ is known as the Planck constant, $\hbar = 1.054 \times 10^{-34} \text{ J} \cdot \text{s}$ is called the reduced Planck constant, and n is the principal quantum number.

3. When an electron transits from a higher energy level to a lower one, a photon will be emitted with an energy

$$E_{\text{photon}} = h\nu = E_i - E_f \tag{5}$$

where E_i and E_f are energies of the initial and final orbitals and ν is the frequency of the emitted photon.

Note that in Bohr's model, the total energy of the orbiting electron is the sum of the kinetic energy due to the motion and the potential energy resulting from the electrical binding from the nucleus.

The hydrogen spectrum can be classified into several different series by the final principal quantum number n . Table 1 lists the five most-known series and their spectral ranges.

Table 1: Series of the emitted spectrum

Principal quantum number after the transition	Series Name	Series range
$n = 1$	Lyman series	Ultraviolet
$n = 2$	Balmer Series	Visible + Ultraviolet
$n = 3$	Paschen Series	Infrared
$n = 4$	Brackett Series	Infrared
$n = 5$	Pfund Series	Infrared

Historically, the spectrum lines in the Balmer series are named sequentially by the Greek letter, as Table 2 shows. The Balmer series can be seen in the solar spectrum. Also, H_{α} is an important line used in astronomy to detect the presence of hydrogen. Moreover, if a spectrometer of high resolution is used to examine the Balmer series, it will be found that they all have closely spaced doublets, known as fine structure.

Table 2: Balmer series

Transition of n	Name	Wavelength (nm) ¹	Energy difference (eV)	Color
3 → 2	H _α	656.3	1.89	Red
4 → 2	H _β	486.1	2.55	Green
5 → 2	H _γ	434.5	2.86	Blue
6 → 2	H _δ	410.2	3.03	Violet
7 → 2	H _ε	397.0	3.13	Ultraviolet
∞ → 2	Balmer break	364.6	3.40	Ultraviolet

In this experiment, a spectrometer is used to study the spectrum of hydrogen atoms. Fig. 1 shows the schematic of the spectrometer. The light to be analyzed enters the collimator through a narrow slit positioned at the focal plane of the collimator lens, which makes the light a thin, parallel beam, ensuring that all the light from the slit strikes the diffracting element at the same angle of incidence. If the beam is composed of many different colors, each color is diffracted to a different angle for a given diffraction order. A telescope can be rotated to collect the diffracted light. The measured diffraction angle can be used to determine the wavelength of the light by

$$d \sin \theta = m\lambda, \quad m \in \text{integer} \tag{6}$$

where d is the line spacing of the grating and m is the diffraction order.

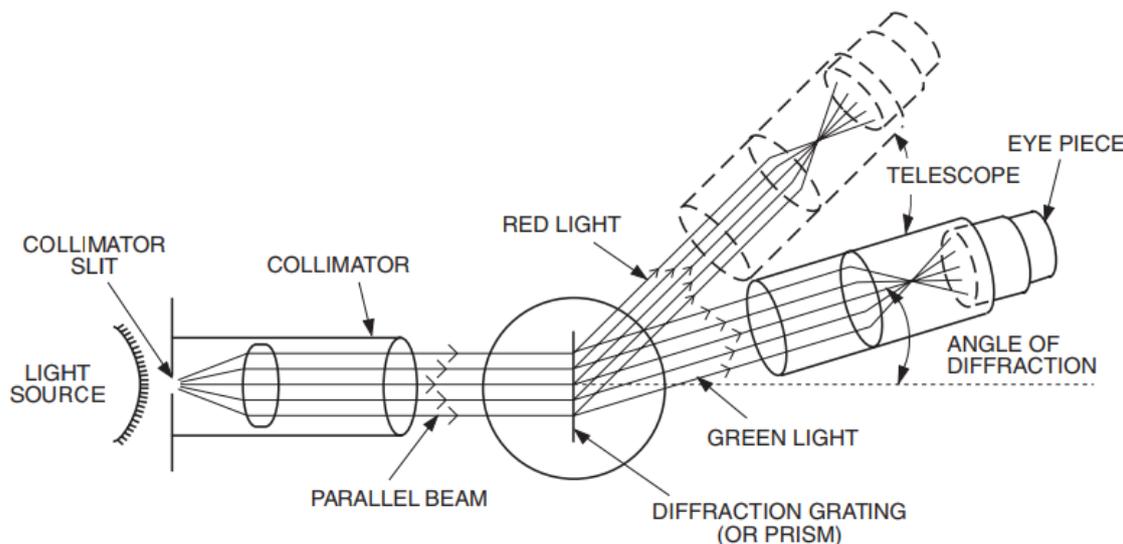


Fig. 1. The diagram of a spectrometer

After the wavelengths of the Balmer series are determined, one can use the result with Bohr’s model to obtain the Planck constant.

The spectrometer can also be used to measure the index of refraction of a prism using the minimum deviation. The angle of deviation for light traversing a prism is shown in Fig. 2. For a given wavelength of light traversing a given prism, there is an angle of incidence for which the angle of deviation is a minimum. This angle depends on the index of refraction of the prism and

¹ The wavelength listed in Table 2 is for the light in the air.

the angle (labeled A in Fig. 2) between the two sides of the prism traversed by light. The equation gives the relationship between these variables:

$$n = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\frac{A}{2}} \tag{7}$$

where n is the index of refraction of the prism, A is the angle between the prism sides traversed by the light, and D is the angle of minimum deviation. Since n varies with wavelength, the angle of minimum deviation also varies. See the procedures in the in-lab activity for the details of the measurement.

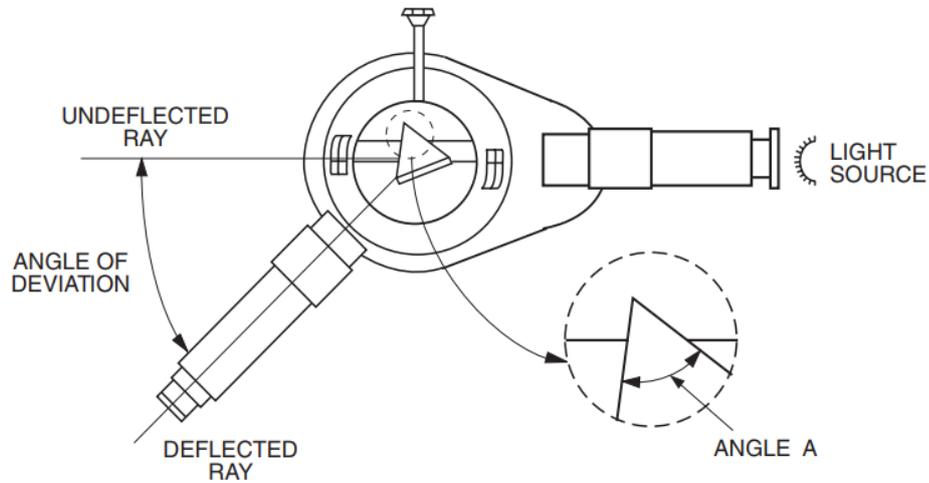


Fig. 2. Angle of deviation

C. Apparatus



Fig. 3. Spectrometer assemble

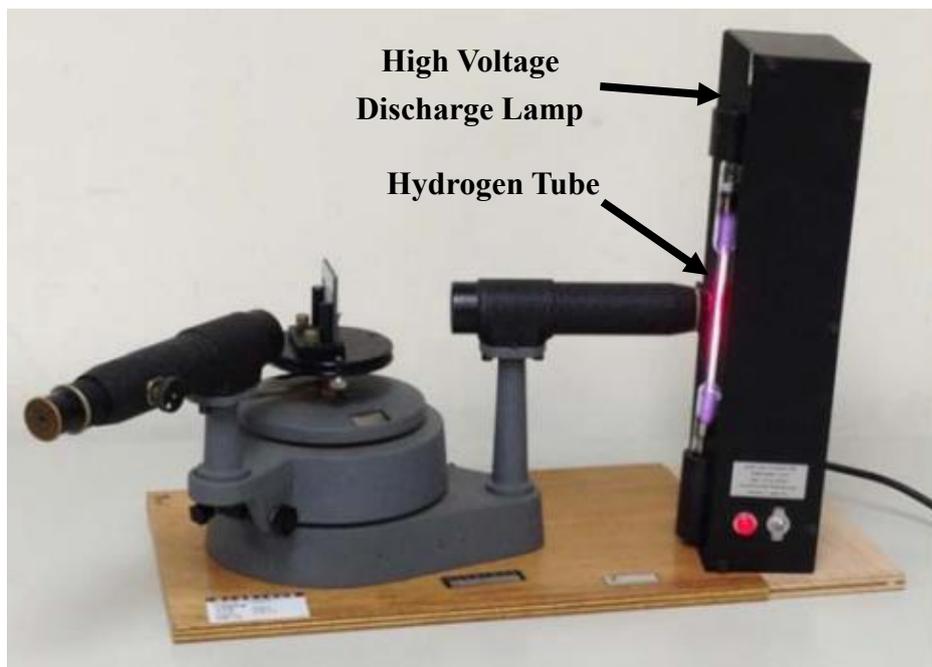


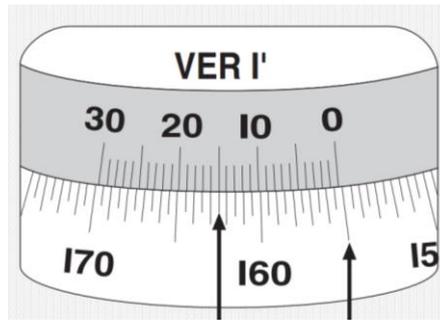
Fig. 4. Experimental setup

Note that the diffraction grating used in this experiment has 600 lines/mm.

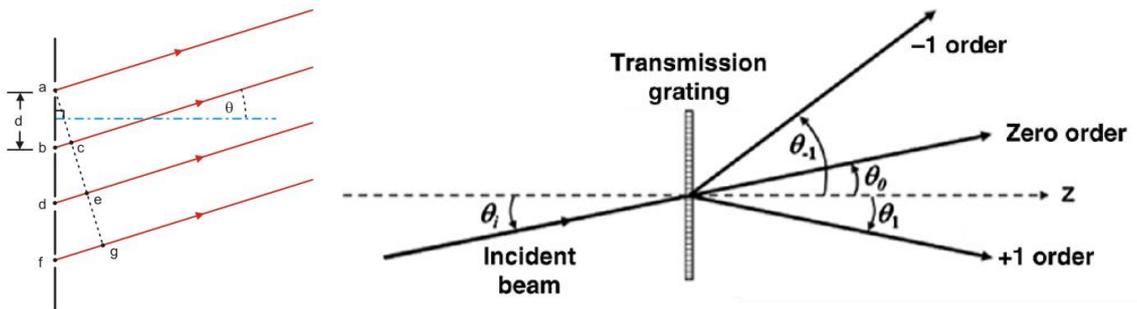
CAUTION: The hydrogen discharge lamp is powered by high voltage, and the tube gets hot. DO NOT touch the tube especially near the ends where the electrical contacts are made.

D. Procedures

1. Pre-lab assignments (hand in before the experiment)
 - (0) See the following link for the basic introduction of this lab:
<https://youtu.be/25Y3AKzx0hM>
 - (1) Make a flowchart of this experiment and answer the questions.
 - (2) From eq(3), (4), and (5), prove that the Bohr model gives the same result as the Ryberg's formula (eq (2)). Also, find the Rydberg constant in terms of the electron mass m , the Planck constant h , and other constants.
 - (3) Following 2, given the wavelength of visible light is between 400 nm and 700 nm, find the wavelengths of the hydrogen spectral lines that are visible.
 - (4) Following 3, in this experiment, you will observe the hydrogen spectrum by a diffraction grating and a spectrometer. Given the diffraction equation (eq(6)), find the diffraction angles for all the spectral lines that you can see through the grating. (Hint: You will see different sets of visible lines for different diffraction orders.)
 - (5) Find the reading of the vernier scales shown in the picture below.



- (6) In eq(6), it's assumed that the incident light is perpendicular to the diffraction grating. This diffraction equation can be derived using the optical path difference between two parallel lights from two adjacent grating lines.



- (i) The figure to the left shows a diffraction grating. Derive eq(6).
- (ii) However, it's impossible to ensure the normal incident condition in the real experiment. Consider the first order of diffraction. How do you modify the diffraction equation if the incident beam is not perpendicular to the grating, as shown in the picture to the right?
- (iii) Suppose the wavelength of the incident beam is 600 nm and the incident angle θ_i is 2 degrees. Use the modified equation to obtain θ_1 and θ_{-1} . Then find the angles of diffracted lights of +1 and -1 order with respect to the zero order. These angles are the angles that can be measured in the real experiments. Finally, use the two angles with eq(6) to determine the "measured" wavelengths of the refracted lights. Compare the average "measured" wavelength with the given wavelength 600 nm.
- (iv) From the results of (iii), in the experiment, while using the eq(6), how do you solve the inclined effect?

2. In-lab activities

- (1) To focus the spectrometer
 - (i) While looking through the telescope, slide the eyepiece in and out until the cross-hairs come into sharp focus.
 - (ii) Focus the telescope at infinity. This is best accomplished by focusing on a distant object. (e.g. out the window)
 - (iii) Check that the collimator slit is partially open (use the slit width adjustment screw).

- (iv) Align the telescope directly opposite the collimator, as Fig. 5 shows.
- (v) Looking through the telescope, adjust the focus of the collimator and, if necessary, the rotation of the telescope until the slit comes into sharp focus. **Do NOT change the focus of the telescope.**
- (vi) Tighten the telescope rotation lock screw, then use the fine adjustment knob to align the vertical line of the graticule with the fixed edge of the slit. If the slit is not vertical, loosen the slit lock ring, realign the slit, and retighten the lock ring. Adjust the slit width for a clear, bright image.
- (vii) Measurements of the diffraction angle are always made with the graticule line aligned along the fixed edge of the slit, so a very narrow slit is not necessarily advantageous.

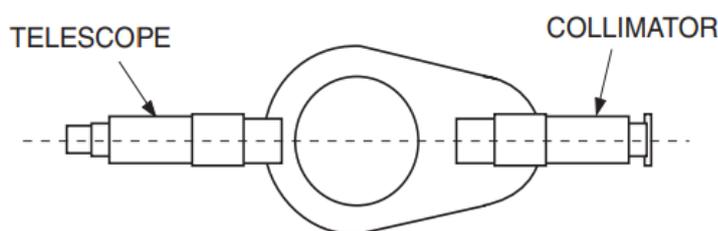


Fig. 5. Align the telescope directly opposite the collimator

- (2) To set up the discharge lamp
 - (i) Place the lamp approximately one centimeter from the slit.
 - (ii) Adjust the height of the lamp so that the objective lens is horizontally aligned with the center of the hydrogen tube.
 - (iii) Adjust the slit width, so the slit image is bright.

Note: A good tube will have a bright red color near its center, and an aged one tube will be more pinkish near the center. For best results, it is important to have a good tube.
- (3) Direct observation of the hydrogen spectrum via the grating
 - (i) Hold the diffraction grating with your hands. Observe the hydrogen spectrum by directly looking through the grating at the discharge tube.
 - (ii) While observing the hydrogen spectrum, rotate the diffraction grating with respect to the axis **(a)** perpendicular and **(b)** parallel to the lines of the grating. Describe your finding in the post-lab report. (Take photos as a record)
- (4) Observe the hydrogen spectrum by the spectrometer and the grating
 - (i) To align the diffraction grating
 - (a) Using the thumbscrews, attach the grating mount so that it is perpendicular to the engraved lines.

- (b) Insert the diffraction grating into the clips of the mount. The grating should spread the colors of the incident light horizontally so that the rotation of the telescope will allow you to see the different colored images of the slit.
- (ii) To observe the spectral lines (Fig. 6)
 - (a) Record the angle reading from the vernier scale for the zero order diffraction. Rotate the telescope to find the bright slit image. Align the vertical cross-hair with the fixed edge of the image and record the angle reading from the vernier scales for each diffracted light.
 - (b) Calculate the diffraction angles for the lines you can see. Note that the diffraction grating diffracts the incident light into identical spectra on both sides of the undiffracted beam. Therefore, you have to measure the spectral lines on both sides. Also, you can use your phone to help you “see” the lines.
 - (c) Compare the measurements with the theoretical values obtained in the pre-lab assignment (Q4).
 - (d) Use the results to obtain Rydberg constant and Planck constant. Calculate the standard deviation of the mean of your measurements and compare the result with the reference value. (Hint: Q2 in the pre-lab assignment)

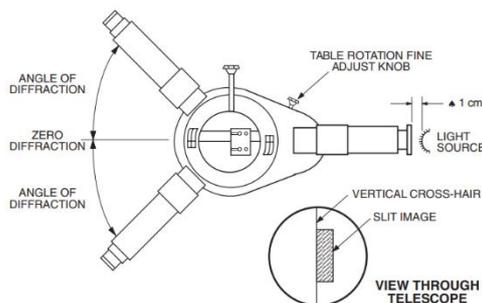


Fig. 6. Observe the light spectrum with the spectrometer

- (5) Observe the hydrogen spectrum with the spectrometer and the prism
 - (i) Remove the mount for the grating and the grating. Place the prism on the spectrometer table.
 - (ii) Observe the hydrogen spectrum. Is it different from the observation via the diffraction grating? Describe the difference in the post-lab report (Take a photo for the record)
 - (iii) To measure the angle of minimum deviation of the prism
 - (a) Determine the direction to which the light is refracted, then align the telescope and spectrometer table base so the slit image can be viewed through the telescope.
 - (b) While looking through the telescope, rotate the prism slightly back and forth. The spectral lines will move when you rotate the prism. Focus on one spectral line while you are observing. Stop at the position where the chosen

line starts to move in the opposite direction. Record the angle from the reading of the vernier scale. Note that during the measurement, the spectrometer table should be fixed. Choose the other spectral lines and re-do the steps.

(c) Remove the prism and rotate the telescope to align the crosshair with the fixed edge of the undiffracted beam. Measure the angle from the vernier scale. The difference between this angle and each recorded angle in (b) is the angle of minimum deviation for each diffracted light.

(d) Use eq(7) to determine the index of refraction of the prism.

3. Post-lab report

- (1) Recopy and organize your data from the in-lab tables in a neat and readable form.
- (2) Analyze the data you obtained in the lab and answer the given questions
- (3) Compare the results with the theory, and discuss the uncertainties in the experiments and how they influence them. (Quantitatively, if possible.)

E. Questions

1. In the theory section, we have assumed that the nucleus mass in Bohr's model is infinitely large. If the mass of the nucleus is now taken into consideration,
 - (1) Find the wavelengths of H_α , H_β , and H_γ spectral lines.
 - (2) If the hydrogen source is replaced with a deuterium source, find the wavelengths of H_α , H_β , and H_γ spectral lines in this case. Also, for each spectral line, calculate the average wavelength $\bar{\lambda}$ and the wavelength difference $\Delta\lambda$ between the hydrogen and deuterium, which is known as the isotope shift.
 - (3) Suppose we have a discharge tube filled with the mixture of hydrogen and deuterium. Discuss whether we can resolve this isotope shift in the experiment.

(Hint: to resolve the spectrum, the minimum wavelength difference $\Delta\lambda_{\min}$ between two adjacent spectral lines is

$$\Delta\lambda_{\min} = \frac{\bar{\lambda}}{mN} \quad (8)$$

where m is the diffraction order and N is the total number of grating lines that are illuminated by the light.)

2. Prove eq(7) under the condition that the cross-section of the prism is an isosceles triangle. (Hint: Consider the geometric symmetry and the reversibility of optical path.)
3. Plot the reciprocal of the wavelength $1/\lambda$ of each emission line found in the experiment versus $1/n_i^2$. Draw a best-fit line through the points on your graph. Use the slope and the y-intercept to determine the Rydberg's constant, respectively. Discuss the results with the average of the calculated Rydberg's constant in the experiment. Also, compare the x-intercept with the theoretical value.

4. Following (3), one can also choose to plot the $1/\lambda$ of each emission line found in the experiment versus $1/4 - 1/n_i^2$ based on the fact that the emitted lights are in the Balmer series. What is the difference between these two plots? Would they give the same results for Rydberg's constant? How would the intercepts change? Explain.
5. How do you determine the index of refraction of the unknown liquid by the apparatus used in this lab? Explain.
6. **(Optional)** Following Q6 in the pre-lab assignment, use Matlab to simulate the inclined effect for $\theta_i = 0^\circ \sim 10^\circ$.

F. References

- Amrani, D. "Hydrogen Balmer series measurements and determination of Rydberg's constant using two different spectrometers." *European Journal of Physics* 35.4 (2014): 045001.
- Mayer, V. V., and E. I. Varaksina. "Normal light dispersion in laboratory experiments." *Physics Education* 57.1 (2021): 015013.